Engr228 **Exam #2** **Autumn, 2016**

NAME KEY SCORE______/30

This test is 60 minutes in length and is closed book except for your calculator and one side of a cheat sheet.

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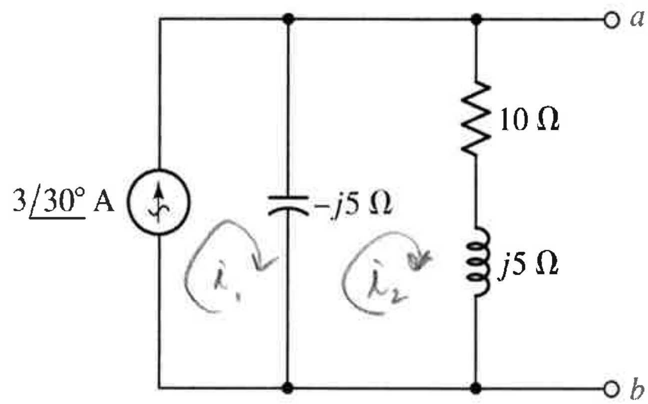
10 pts Phasor Analysis

7 pts Find the current through the 10Ω resistor. Express your answer in polar form.



Current Divider

$$i_{10\Omega} = \frac{3\angle 30^\circ(-j5)}{-j5+j5+10}$$

$$= \frac{3\angle 30^\circ\,(5\angle -90^\circ)}{10}$$

$$i_{10\Omega} = 1.5\angle -60^\circ \text{ A}$$

Mesh

$$i_1 = 3\angle 30^\circ$$

mesh i_2: $-j5(i_2 - i_1) + 10 i_2 + j5 i_2 = 0$

$$i_2 = \frac{-15\angle 30^\circ}{-j5+j5+10} = 1.5\angle -60^\circ \text{ A}$$

3 pts Find the voltage across the inductor. Express your answer in sinusoidal form.

$$V_{IND} = i_{IND} Z_{IND}$$

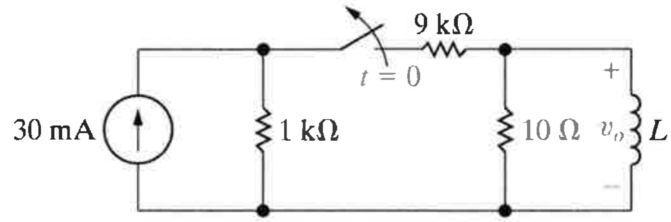
$$= (1.5\angle -60^\circ)(5\angle 90^\circ)$$

$$= 7.5\angle 30^\circ \text{ V}$$

$$= 7.5\cos(\omega t + 30^\circ) \text{ V}$$

10 pts First Order Circuits

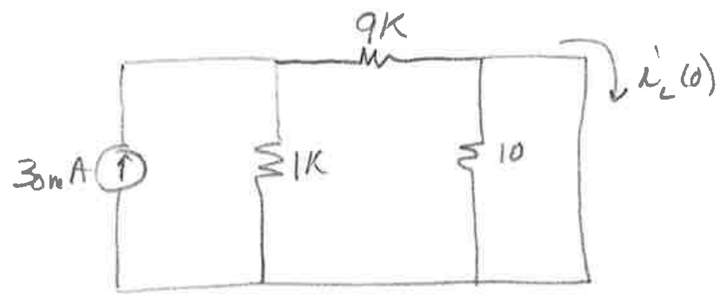
4 pts The DC current source has been applied for a long time before the switch opens at t = 0 seconds. Find $I_L(0)$ i.e. the initial inductor current for L = 100mH.



using current divider:

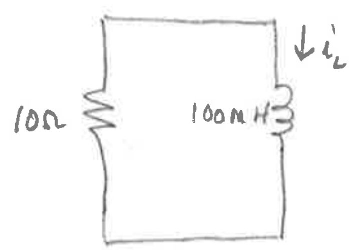
$$i_L(0) = \frac{30mA(1K)}{1K+9K}$$

$$\boxed{i_L(0) = 3mA}$$



2 pts What is the time constant (τ) of this circuit for t > 0 seconds?

$$\tau = \frac{L}{R} = \frac{.1}{10} = \boxed{10mS}$$



2 pts What is I_L as time goes to infinity, i.e. $I_L(\infty)$?

$$i_L(\infty) = 0 \text{ A}$$

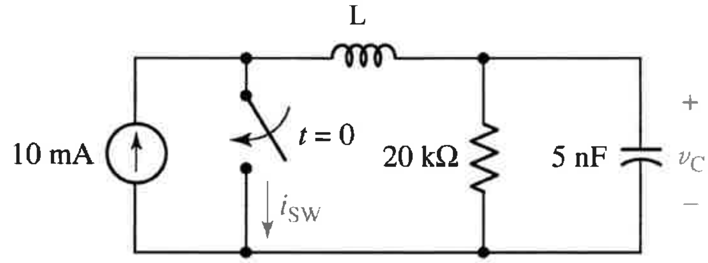
2 pts Find $I_L(t)$ for t ≥ 0 seconds

$$i_L(t) = i_L(0)\, e^{-\frac{t}{\tau}}$$

$$\boxed{i_L(t) = 3e^{-100t} \text{ mA} \quad t \geq 0}$$

10 pts Second Order Circuits

The switch in the figure at the right has been open for a long time and closes at t = 0 seconds. Obtain an expression for the voltage labeled $v_C(t)$ for $t \geq 0$ seconds. Note that the correct answer has *transient terms only*. Take L = 8.0 Henries.



1) FIND I.C.'s

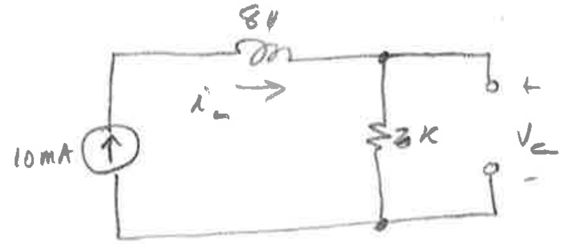
$i_L(0) = 10mA$

$V_C(0) = (10mA)(20K) = 200V$

2) FIND α and ω_0

$\alpha = \frac{1}{2RC} = 5000$ Rad/sec

$\omega_0 = \frac{1}{\sqrt{LC}} = 5000$ Rad/sec



$\alpha = \omega_0$ So system is critically damped

$V_c(t) = A_1 t e^{s_1 t} + A_2 e^{s_2 t}$

$s_1 = s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5000$

$V_c(t) = (A_1 t + A_2) e^{-5000t}$

3) FIND A_1 AND A_2

@ $t = 0$, $V_c = 200V \Rightarrow 200 = A_2$ So $V_c(t) = (A_1 t + 200) e^{-5000t}$

writing a KCL equation @ $t=0$;

$i_L + i_R + i_C = 0$

$-10mA + \frac{200}{20K} + C\frac{dV_c}{dt} = 0$

$-10mA + 10mA + 5\times10^{-9}\left[A_1 t(-5000)e^{-5000t} + e^{-5000t}A_1 + 200(-5000)e^{-5000t}\right]\Big|_{t=0} = 0$

$A_1 + 200(-5000) = 0 \Rightarrow A_1 = 1000000$

$\therefore V_c(t) = (1{,}000{,}000t + 200)e^{-5000t}$ V